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2005 J. Phys. A: Math. Gen. 38 2525

(http://iopscience.iop.org/0305-4470/38/11/013)

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS A: MATHEMATICAL AND GENERAL

J. Phys. A: Math. Gen. **38** (2005) 2525–2541 doi:10.1088/0305-4470/38/11/013

Bivalued 'click'–'no-click' probabilities for EPRB spin correlations

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Received 1 October 2004, in final form 14 December 2004
Published 2 March 2005
Online at stacks.iop.org/JPhysA/38/2525

Abstract

We derive non-local bivalued probabilities of *click* or *no-click* outcomes—corresponding respectively to *detection* or *no-detection* of the particles—where each of the Einstein–Podolsky–Rosen–Bohm (EPRB) pair of particles is observed in only one of the $(2s+1)$ possible spin projection channels. In the case of observations in maximum *spin down* channels, our results coincide exactly with those given by Wódkiewicz [3]. We analyse the nature of correlation between the *click–no-click* results with the help of non-local conditional probabilities and information entropies. We observe that the violation of co-planar BC inequalities is essentially due to stronger correlations between the *click–no-click* outcomes in the spin projection channels $|\lambda_a| = |\lambda_b|$. This observation is also supported by spin transmission Bell inequalitites.

PACS numbers: 03.65.Ud, 03.67.−a

1. Introduction

Quantum entanglement, involving a pair of spatially separated (non-interacting) particles, described through a non-separable, joint quantum state has been a major source of discussion—initiated with the famous Einstein–Podolsky–Rosen (EPR) argument [1]—in the conceptual development of quantum theory. The EPR paper stimulated numerous discussions concerning the fundamental differences between quantum and classical concepts. It has been realized [2] that quantum correlations do not obey the classically satisfactory *local realistic* theories. Non-locality underlying quantum entanglement has thus occupied the forefront of discussions. The (EPRB) entanglement involving spin s particles had been analysed by Wódkiewicz [3], using non-local (analyser dependent) bivalued probabilities of dichotomic variables '1' and '0' corresponding, respectively, to *click* and *no-click* outcomes at the detectors. Wódkiewicz [3] employed the spin projection operators $\hat{P}(\vec{a})$, $\hat{P}(\vec{b})$, at the analyser orientations $\vec{a}$ and $\vec{b}$, which correspond to *click* $\rightarrow$ 1 or *no-click* $\rightarrow$ 0 outcomes for the particles, as the statistical variates and the joint transmission $p(\vec{a};\vec{b})$ given by

$$p(\vec{a};\vec{b}) = \langle\psi_{\text{EPRB}}|\hat{P}(\vec{a})\otimes\hat{P}(\vec{b})|\psi_{\text{EPRB}}\rangle \tag{1}$$

are identified to be an average[3] of the dichotomic variables 1, 0 in terms of non-local bivalued probabilities. According to local hidden variable theories the joint spin transmissions are constrained by a Bell type inequality [4]

$$-1 \leqslant p(\vec{a};\vec{b}') + p(\vec{a}';\vec{b}') + p(\vec{a}';\vec{b}) - p(\vec{a};\vec{b}) - p(\vec{a}) - p(\vec{b}) \leqslant 0, \tag{2}$$

where $\vec{a}, \vec{a}', \vec{b}, \vec{b}'$ denote four different orientations of the analysers. The spin transmission Bell inequality of form (2) has been shown [3, 5] to be valid for any arbitrary spin s. However, the projection operators used by Wódkiewicz correspond to *maximum down* spin projections of particles 1, 2, defined with respect to the analyser orientations $\vec{a}$ and $\vec{b}$ respectively. Therefore, the outcomes, namely *click* (*no-click*) correspond to *detection (no-detection)* of the particles, in the *maximum spin down* projection channels alone. We emphasize that the non-locality underlying random bivalued outcomes is not restricted to *maximum spin down* channels alone. The bivalued probabilities associated with the random dichotomic outcomes from all the other possible spin channels exhibit characteristic specific dependences on the orientations of the spin analysers and hence exhibit non-locality. It would be interesting to analyse how these strange quantum correlations allow for a smooth transition to the classical domain in the large spin limit $s \rightarrow \infty$. Projection $\lambda = s\cos\theta$, of a classical angular momentum vector of magnitude s, can vary between $-s$ to $+s$ continuously and the study of statistical correlations between the measurements of various spin projections of a pair of angular momentum vectors associated with the EPRB spin-s singlet state provides a framework for understanding the underlying non-local randomness. The purpose of this paper is to study the non-local correlations between the random *click–no-click* results in all possible spin projection channels.

In section 2 we derive the joint distributions of dichotomic random variables $\varepsilon_a, \varepsilon_b$ ($\varepsilon_a, \varepsilon_b = 1, 0$), which correspond to the *click–no-click* outcomes in each of the spin projection channels λ_a, λ_b. In the maximum *down* spin projection channel, our results coincide exactly with those of Wódkiewicz. The *click–no-click* results in different spin projection channels are analysed using non-local conditional probabilities.

We examine the behaviour of the conditional probabilities in the classical limit $s \rightarrow \infty$. This analysis helps us to realize that the EPRB non-local *click–no-click* correlations turn smoothly into correlations between classical antiparallel angular momentum vectors in the classical limit.

How much information is revealed by these *click–no-click* random outcomes? We address this question in section 3 with the help of Gibbs–Shannon information entropies [6, 7] constructed using bivalued probabilities. It has been realized [7, 8] that mutual information entropy serves as a quantitative measure of entanglement. We study the strength of correlations (using normalized mutual information entropies) in different spin projection channels λ_a, λ_b. We observe that $|\lambda_a| = |\lambda_a|$ channels record stronger correlations between *click–no-click* outcomes. This result reflects itself in the information theoretic Braunstein–Caves [9] inequalities: Co-planar BC inequalities are violated only for the outcomes in the channels $|\lambda_a| = |\lambda_b|$. Moreover, we show that the spin transmission Bell inequality given by equation (2) also reproduces the same result. Section 4 contains some concluding remarks.

[3] Here, $|\psi_{\text{EPRB}}\rangle = \frac{1}{\sqrt{(2s+1)}}\sum_{\lambda=-s}^{s}(-1)^{s-\lambda}|\lambda;-\lambda\rangle$ denotes EPRB entangled spin singlet of two spin-s particles.

2. Non-local bivalued probabilities for EPRB correlations

We consider projection operators $\hat{P}^1_{\lambda_a}(\vec{a})$, $\hat{P}^2_{\lambda_b}(\vec{b})$ (the indices 1, 2 refer to particles 1 and 2 respectively) corresponding to different spin projections, $\lambda_a, \lambda_b = s, s-1, \ldots, -s$, along the analyser orientations $\vec{a}, \vec{b}$, given by

$$\begin{aligned}\hat{P}^1_{\lambda_a}(\vec{a}) &= |\vec{a}, \lambda_a\rangle\langle\vec{a}, \lambda_a| \\ \hat{P}^2_{\lambda_b}(\vec{b}) &= |\vec{b}, \lambda_b\rangle\langle\vec{b}, \lambda_b|.\end{aligned} \tag{3}$$

Here, $|\vec{a}, \lambda_a\rangle$, $|\vec{b}, \lambda_b\rangle$ denote the rotated spin states:

$$\begin{aligned}|\vec{a}, \lambda_a\rangle &= \hat{R}(\phi_a, \theta_a, 0)|z, \lambda_a\rangle = \sum_{\lambda_1=-s}^{s} D^s_{\lambda_1\lambda_a}(\phi_a, \theta_a, 0)|z, \lambda_1\rangle, \\ |\vec{b}, \lambda_b\rangle &= \hat{R}(\phi_b, \theta_b, 0)|z, \lambda_b\rangle = \sum_{\lambda_2=-s}^{s} D^s_{\lambda_2\lambda_b}(\phi_a, \theta_a, 0)|z, \lambda_2\rangle,\end{aligned} \tag{4}$$

where $|z, \lambda_1\rangle$, $|z, \lambda_2\rangle$ denote the spin states with a common laboratory z-axis as the quantization axis; $D^s(\alpha, \beta, \gamma)$ denotes $(2s+1)$-dimensional irreducible representation of rotations and α, β, γ are the Euler angles of rotation [10].

The bivariate characteristic function [11], with the projection operators $\hat{P}^1_{\lambda_a}(\vec{a})$ and $\hat{P}^2_{\lambda_b}(\vec{b})$ as statistical variates, is defined by the quantum mechanical average

$$\phi(r_a, r_b) = \left\langle\left(\mathrm{e}^{\mathrm{i}\hat{P}^1_{\lambda_a}(\vec{a})r_a} \otimes \mathrm{e}^{\mathrm{i}\hat{P}^2_{\lambda_b}(\vec{b})r_b}\right)\right\rangle = \mathrm{Tr}\left[\hat{\rho}\left(\mathrm{e}^{\mathrm{i}\hat{P}^1_{\lambda_a}(\vec{a})r_a} \otimes \mathrm{e}^{\mathrm{i}\hat{P}^2_{\lambda_b}(\vec{b})r_b}\right)\right], \tag{5}$$

where $\hat{\rho} = |\psi_{\mathrm{EPRB}}\rangle\langle\psi_{\mathrm{EPRB}}|$ is the density operator characterizing the EPRB spin-s correlations. From equations (3) and (4) it can be observed that

$$\begin{aligned}\hat{P}^1_{\lambda_a}(\vec{a}) &= \hat{R}(\phi_a, \theta_a, 0)\hat{P}^1_{\lambda_a}(z)\hat{R}^\dagger(\phi_a, \theta_a, 0), \\ \hat{P}^2_{\lambda_b}(\vec{b}) &= \hat{R}(\phi_b, \theta_b, 0)\hat{P}^2_{\lambda_b}(z)\hat{R}^\dagger(\phi_b, \theta_b, 0),\end{aligned} \tag{6}$$

where $\hat{P}^1_{\lambda_a}(z) = |z, \lambda_a\rangle < z, \lambda_a|$ and $\hat{P}^2_{\lambda_b}(z) = |z, \lambda_b\rangle < z, \lambda_b|$. Substituting equation (6) into equation (5) we get

$$\begin{aligned}\phi(r_a, r_b) &= \mathrm{Tr}\left[\hat{\rho}\left(\hat{R}(\phi_a, \theta_a, 0)\,\mathrm{e}^{\mathrm{i}\hat{P}^1_{\lambda_a}(z)r_a}\hat{R}^\dagger(\phi_a, \theta_a, 0)\right)\right. \\ &\qquad \left.\otimes\left(\hat{R}(\phi_b, \theta_b, 0)\,\mathrm{e}^{\mathrm{i}\hat{P}^2_{\lambda_b}(z)r_b}\hat{R}^\dagger(\phi_b, \theta_b, 0)\right)\right] \\ &= \mathrm{Tr}\left[\left(\hat{R}^\dagger(\vec{a}, \vec{b})\hat{\rho}\hat{R}(\vec{a}, \vec{b})\right)\left(\mathrm{e}^{\mathrm{i}\hat{P}^1_{\lambda_a}(z)r_a} \otimes \mathrm{e}^{\mathrm{i}\hat{P}^2_{\lambda_b}(z)r_b}\right)\right] \\ &= \sum_{\lambda_1,\lambda_2,\lambda'_1,\lambda'_2=-s}^{s} (\hat{R}^\dagger(\vec{a}, \vec{b})\hat{\rho}\hat{R}(\vec{a}, \vec{b}))_{\lambda'_1\lambda'_2;\lambda_1\lambda_2}\left(\mathrm{e}^{\mathrm{i}\hat{P}^1_{\lambda_a}(z)r_a}\right)_{\lambda_1\lambda'_1}\left(\mathrm{e}^{\mathrm{i}\hat{P}^2_{\lambda_b}(z)r_b}\right)_{\lambda_2\lambda'_2},\end{aligned} \tag{7}$$

where we have made use of the property of the direct products, $AB \otimes CD = (A \otimes B)(C \otimes D)$ and the cyclic property $\mathrm{Tr}(AB) = \mathrm{Tr}(BA)$ of the traces. Here, $\hat{R}(\vec{a}, \vec{b}) = \hat{R}(\phi_a, \theta_a, 0) \otimes \hat{R}(\phi_b, \theta_b, 0)$. Further, making use of the property $P^2 = P$ of the projection operators, we get

$$\begin{aligned}\left(\mathrm{e}^{\mathrm{i}\hat{P}^1_{\lambda_a}(z)r_a}\right)_{\lambda_1\lambda'_1} &= \delta_{\lambda'_1\lambda_1} + \delta_{\lambda'_1\lambda_a}\delta_{\lambda_1\lambda_a}(\mathrm{e}^{\mathrm{i}r_a} - 1), \\ \left(\mathrm{e}^{\mathrm{i}\hat{P}^2_{\lambda_b}(z)r_b}\right)_{\lambda_2\lambda'_2} &= \delta_{\lambda_2\lambda'_2} + \delta_{\lambda'_2\lambda_b}\delta_{\lambda_2\lambda_b}(\mathrm{e}^{\mathrm{i}r_b} - 1).\end{aligned} \tag{8}$$

Substituting equation (8) into equation (7), we obtain, after rearranging the terms,

$$\phi(r_a, r_b) = \sum_{\varepsilon_a,\varepsilon_b=0,1} \mathcal{P}^s_{\lambda_a\lambda_b}(\varepsilon_a, \varepsilon_b)\,\mathrm{e}^{\mathrm{i}(\varepsilon_a r_a + \varepsilon_b r_b)}, \tag{9}$$

where the characteristic function $\phi(r_a, r_b)$ is cast in its standard statistical form [11] and $\mathcal{P}^s_{\lambda_a\lambda_b}(\varepsilon_a, \varepsilon_b)$, $\varepsilon_a, \varepsilon_b = 0, 1$, are the bivalued probabilities

$$\begin{aligned}
\mathcal{P}^s_{\lambda_a\lambda_b}(1,1) &= (\hat{R}^\dagger(\vec{a},\vec{b})\hat{\rho}\hat{R}(\vec{a},\vec{b}))_{\lambda_a\lambda_b;\lambda_a\lambda_b}\\
\mathcal{P}^s_{\lambda_a\lambda_b}(1,0) &= \sum_{\lambda_2=-s}^{s}(\hat{R}^\dagger(\vec{a},\vec{b})\hat{\rho}\hat{R}(\vec{a},\vec{b}))_{\lambda_a\lambda_2;\lambda_a\lambda_2} - (\hat{R}^\dagger(\vec{a},\vec{b})\hat{\rho}\hat{R}(\vec{a},\vec{b}))_{\lambda_a\lambda_b;\lambda_a\lambda_b}\\
\mathcal{P}^s_{\lambda_a\lambda_b}(0,1) &= \sum_{\lambda_1=-s}^{s}(\hat{R}^\dagger(\vec{a},\vec{b})\hat{\rho}\hat{R}(\vec{a},\vec{b}))_{\lambda_1\lambda_b;\lambda_1\lambda_b} - (\hat{R}^\dagger(\vec{a},\vec{b})\hat{\rho}\hat{R}(\vec{a},\vec{b}))_{\lambda_a\lambda_b;\lambda_a\lambda_b}\\
\mathcal{P}^s_{\lambda_a\lambda_b}(0,0) &= 1 - \sum_{\lambda_1=-s}^{s}(\hat{R}^\dagger(\vec{a},\vec{b})\hat{\rho}\hat{R}(\vec{a},\vec{b}))_{\lambda_1\lambda_b;\lambda_1\lambda_b} - \sum_{\lambda_2=-s}^{s}(\hat{R}^\dagger(\vec{a},\vec{b})\hat{\rho}\hat{R}(\vec{a},\vec{b}))_{\lambda_a\lambda_2;\lambda_a\lambda_2}\\
&\quad + (\hat{R}^\dagger(\vec{a},\vec{b})\hat{\rho}\hat{R}(\vec{a},\vec{b}))_{\lambda_a\lambda_b;\lambda_a\lambda_b}.
\end{aligned} \tag{10}$$

We identify that [12]

$$(\hat{R}^\dagger(\vec{a},\vec{b})\hat{\rho}\hat{R}(\vec{a},\vec{b}))_{\lambda_a\lambda_b;\lambda_a\lambda_b} = |\langle\vec{a},\lambda_a;\vec{b},\lambda_b|\psi_{\text{EPRB}}\rangle|^2 = \frac{1}{(2s+1)}|d^s_{-\lambda_b,\lambda_a}(\theta_{ab})|^2 \tag{11}$$

where $d^s(\theta_{ab})$ denotes rotation about the y-axis, which is perpendicular to the plane containing $\vec{a}$ and $\vec{b}$; θ_{ab} is the angle between $\vec{a}$ and $\vec{b}$. Now, one can obtain, with the help of the unitarity of d matrices,

$$\begin{aligned}
\sum_{\lambda_1=-s}^{s}(\hat{R}^\dagger(\vec{a},\vec{b})\hat{\rho}\hat{R}(\vec{a},\vec{b}))_{\lambda_1\lambda_b;\lambda_1\lambda_b} &= \frac{1}{(2s+1)}\sum_{\lambda_1=-s}^{s}\left|d^s_{-\lambda_b,\lambda_1}(\theta_{ab})\right|^2 = \frac{1}{(2s+1)}\\
\sum_{\lambda_2=-s}^{s}(\hat{R}^\dagger(\vec{a},\vec{b})\hat{\rho}\hat{R}(\vec{a},\vec{b}))_{\lambda_a\lambda_2;\lambda_a\lambda_2} &= \frac{1}{(2s+1)}\sum_{\lambda_2=-s}^{s}\left|d^s_{-\lambda_2,\lambda_a}(\theta_{ab})\right|^2 = \frac{1}{(2s+1)}.
\end{aligned} \tag{12}$$

Substituting equations (11) and (12) into (10), the bivalued probabilities assume a simple form:

$$\begin{aligned}
\mathcal{P}^s_{\lambda_a\lambda_b}(1,1) &= \frac{1}{(2s+1)}|d^s_{-\lambda_b,\lambda_a}(\theta_{ab})|^2\\
\mathcal{P}^s_{\lambda_a\lambda_b}(1,0) = \mathcal{P}^s_{\lambda_a\lambda_b}(0,1) &= \frac{1}{(2s+1)}\left(1-\left|d^s_{-\lambda_b,\lambda_a}(\theta_{ab})\right|^2\right)\\
\mathcal{P}^s_{\lambda_a\lambda_b}(0,0) &= \frac{1}{(2s+1)}\left((2s-1)+\left|d^s_{-\lambda_b,\lambda_a}(\theta_{ab})\right|^2\right).
\end{aligned} \tag{13}$$

Note that the bivalued probabilities are non-local (analyser dependent) and correspond to the following situations:

(i) both the particles are detected ($\varepsilon_a = 1, \varepsilon_b = 1$),
(ii) only particle 1 is detected ($\varepsilon_a = 1, \varepsilon_b = 0$),
(iii) only particle 2 is detected ($\varepsilon_a = 0, \varepsilon_b = 1$),
(iv) both the particles are not detected ($\varepsilon_a = 0, \varepsilon_b = 0$),

and this detection is confined to the spin projections λ_a, λ_b of particles 1, 2 respectively[4].

It can be readily verified from equation (13) that the probabilities $\mathcal{P}^s_{\lambda_a\lambda_b}(\varepsilon_a, \varepsilon_b)$ are normalized i.e., $\sum_{\varepsilon_a,\varepsilon_b=0,1}\mathcal{P}^s_{\lambda_a\lambda_b}(\varepsilon_a, \varepsilon_b) = 1$. The marginal probabilities $\mathcal{P}^s_{\lambda_a}(\varepsilon_a) =$

[4] The *no-click* at the detectors indicate several other possibilities of detection channels available for the particle than the one under observation.

$\sum_{\varepsilon_b=0,1} \mathcal{P}^s_{\lambda_a\lambda_b}(\varepsilon_a, \varepsilon_b)$ and $\mathcal{P}^s_{\lambda_b}(\varepsilon_b) = \sum_{\varepsilon_a=0,1} \mathcal{P}^j_{\lambda_a\lambda_b}(\varepsilon_a, \varepsilon_b)$ are given by

$$\mathcal{P}^s_{\lambda_a}(1) = \frac{1}{2s+1} = \mathcal{P}^s_{\lambda_b}(1), \qquad \mathcal{P}^s_{\lambda_a}(0) = \frac{2s}{2s+1} = \mathcal{P}^s_{\lambda_b}(0), \tag{14}$$

and they correspond to *yes* or *no* events for the isolated observations on each of the particles. We observe that these marginal probabilities are independent of the detection channels λ_a, λ_b and coincide exactly with the *single folded distributions* of Wódkiewicz [3].

The conditional probabilities $\mathcal{P}^s_{\lambda_a\lambda_b}(\varepsilon_a|\varepsilon_b) = \frac{\mathcal{P}^s_{\lambda_a,\lambda_b}(\varepsilon_a,\varepsilon_b)}{\mathcal{P}^s_{\lambda_b}(\varepsilon_b)}$ give the probability of the outcome $\varepsilon_a = 1, 0$ (*yes or no*) in the channel λ_a for particle 1, under the condition that particle 2 has given rise to the result $\varepsilon_b = 1$ or 0 (*yes or no*) in the detection channel λ_b, and are given by

$$\begin{aligned}
\mathcal{P}^s_{\lambda_a\lambda_b}(1|1) &= \left|d^s_{-\lambda_b,\lambda_a}(\theta_{ab})\right|^2 \\
\mathcal{P}^s_{\lambda_a\lambda_b}(1|0) &= \frac{1}{2s}\left(1 - \left|d^s_{-\lambda_b,\lambda_a}(\theta_{ab})\right|^2\right) \\
\mathcal{P}^s_{\lambda_a\lambda_b}(0|1) &= 1 - \left|d^s_{-\lambda_b,\lambda_a}(\theta_{ab})\right|^2 \\
\mathcal{P}^s_{\lambda_a\lambda_b}(0|0) &= \frac{1}{2s}\left((2s-1) + \left|d^s_{-\lambda_b,\lambda_a}(\theta_{ab})\right|^2\right).
\end{aligned} \tag{15}$$

Observing now that $d^s_{s,-s}(\theta_{ab}) = (-1)^{2s}\left(\sin\frac{\theta_{ab}}{2}\right)^{2s}$, we obtain the following conditional probabilities:

$$\begin{aligned}
\mathcal{P}^s_{-s-s}(1|1) &= \left(\sin\frac{\theta_{ab}}{2}\right)^{4s} & \mathcal{P}^s_{-s-s}(1|0) &= \frac{1}{2s}\left[1 - \left(\sin\frac{\theta_{ab}}{2}\right)^{4s}\right] \\
\mathcal{P}^s_{-s-s}(0|1) &= 1 - \left(\sin\frac{\theta_{ab}}{2}\right)^{4s} & \mathcal{P}^s_{-s-s}(0|0) &= \frac{1}{2s}\left[(2s-1) + \left(\sin\frac{\theta_{ab}}{2}\right)^{4s}\right],
\end{aligned} \tag{16}$$

for the bivalued outcomes in the *maximum spin down* channels i.e., for $\lambda_a = \lambda_b = -s$. It can be readily verified that the conditional probabilities given by Wódkiewicz [3] agree with the above results. For the spin channel $\lambda_a = s, \lambda_b = -s$, the conditional probabilities are given by

$$\begin{aligned}
\mathcal{P}^s_{-s-s}(1|1) &= \left(\cos\frac{\theta_{ab}}{2}\right)^{4s} & \mathcal{P}^s_{-s-s}(1|0) &= \frac{1}{2s}\left[1 - \left(\cos\frac{\theta_{ab}}{2}\right)^{4s}\right] \\
\mathcal{P}^s_{-s-s}(0|1) &= 1 - \left(\cos\frac{\theta_{ab}}{2}\right)^{4s} & \mathcal{P}^s_{-s-s}(0|0) &= \frac{1}{2s}\left[(2s-1) + \left(\cos\frac{\theta_{ab}}{2}\right)^{4s}\right],
\end{aligned} \tag{17}$$

where we have used $d^s_{-s,-s}(\theta_{ab}) = \left(\cos\frac{\theta_{ab}}{2}\right)^{2s}$.

We further note that as a result of the symmetry [10] $d^s_{\lambda_b,\lambda_a} = (-1)^{\lambda_b-\lambda_a} d^s_{\lambda_a,\lambda_b} = (-1)^{\lambda_b-\lambda_a} d^s_{-\lambda_b,-\lambda_a}$, of the d matrices, our bivalued conditional probabilities obey the following properties:

$$\mathcal{P}^s_{\lambda_a\lambda_b}(\varepsilon_a|\varepsilon_b) = \mathcal{P}^s_{\lambda_b\lambda_a}(\varepsilon_a|\varepsilon_b) = \mathcal{P}^s_{-\lambda_a-\lambda_b}(\varepsilon_a|\varepsilon_b) = \mathcal{P}^s_{-\lambda_b-\lambda_a}(\varepsilon_a|\varepsilon_b). \tag{18}$$

This immediately tells us that the results of measurements in the spin projection channels (λ_a, λ_b), (λ_b, λ_a), $(-\lambda_a, -\lambda_b)$ and $(-\lambda_b, -\lambda_a)$ are all identical.

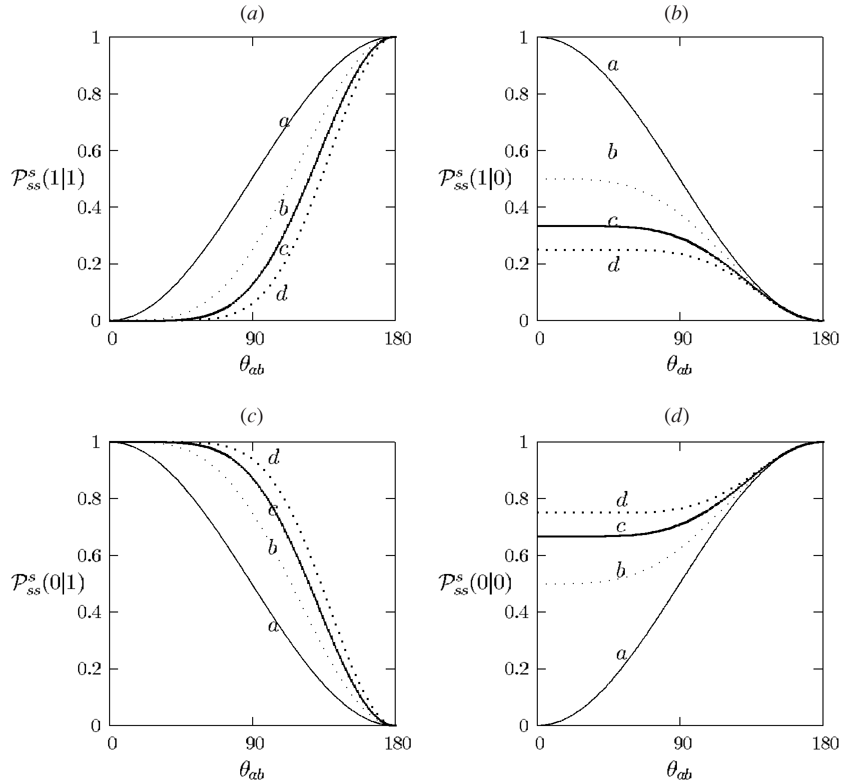

Figure 1. Conditional probabilities of the bivalued outcomes in the spin projection channels $\lambda_a = s$, $\lambda_b = s$, as a function of the analyser orientation angle θ_{ab} for different spin values. Curve *a*: spin-$\frac{1}{2}$, curve *b*: spin-1, curve *c*: spin-$\frac{3}{2}$ and curve *d*: spin-2.

We have plotted the conditional probabilities $\mathcal{P}^s_{s\pm s}(\varepsilon_a|\varepsilon_b)$ for spin values $s = \frac{1}{2}, 1, \frac{3}{2}, 2$ and $\mathcal{P}^s_{s0}(\varepsilon_a|\varepsilon_b)$,[5] for spin values $s = 1, 2, 3, 4$, as a function of the angle θ_{ab} in figures 1–3.

We observe from figures 1 and 2 that as the spin value s increases, $\mathcal{P}^s_{s\pm s}(1|0)$ tends to 0 (see figures 1(b) and 2(b)) and $\mathcal{P}^s_{s\pm s}(0|0)$ tends to 1 (see figures 1(d) and 2(d)) for all angles θ_{ab}, indicating that in the classical limit $s \to \infty$, the *click* and *no-click* outcomes for particle 1 in the spin channels ($\lambda_a = s$, $\lambda_b = \pm s$) are, respectively, 0% and 100% certain, when particle 2 has been realized to register a *no-click* result. Further, it is clear from figures 1(*a*) and (*c*) that in the classical limit $s \to \infty$

$$\mathcal{P}^s_{ss}(1|1) = \begin{cases} 0 & \text{for all angles } \theta_{ab} \neq 180^\circ, \\ 1 & \text{for } \theta_{ab} = 180^\circ; \end{cases}$$

$$\mathcal{P}^s_{ss}(0|1) = \begin{cases} 1, & \text{for all angles } \theta_{ab} \neq 180^\circ, \\ 0, & \text{for } \theta_{ab} = 180^\circ; \end{cases}$$

indicating that when particle 2 registers a *click* in the channel $\lambda_b = s$, particle 1 *always* registers a *no-click* result and *never* gives a *click* outcome in the $\lambda_a = s$ channel, for all the range of analyser orientations except for the anti-parallel orientation namely, $\theta_{ab} = 180^\circ$. A perfect correlation ($\mathcal{P}^s_{ss}(1|1) \to 1$ for anti-parallel analyser orientations) is realized in the

[5] We have calculated $\mathcal{P}^s_{s0}(\varepsilon_a|\varepsilon_b)$ using $d^s_{0s}(\theta_{ab}) = (-1)^{2s}\frac{\sqrt{2s!}}{s!}(\cos\frac{\theta_{ab}}{2}\sin\frac{\theta_{ab}}{2})^s$ in equation (15).

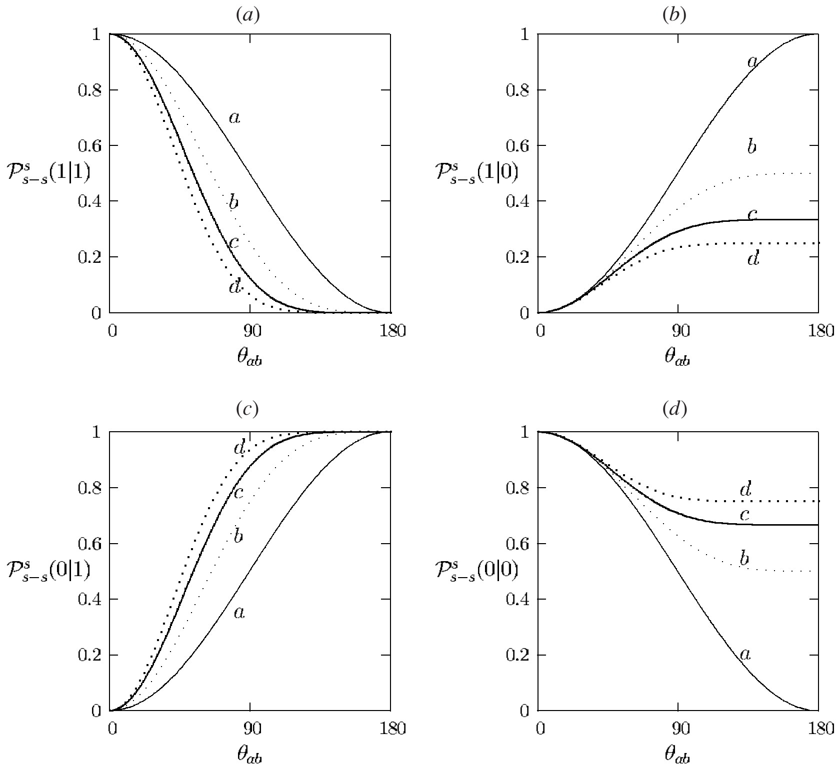

Figure 2. Conditional probabilities of the bivalued outcomes in the spin projection channels $\lambda_a = s, \lambda_b = -s$, as a function of the analyser orientation angle θ_{ab} for different spin values. Curve *a*: spin-$\frac{1}{2}$, curve *b*: spin-1, curve *c*: spin-$\frac{3}{2}$ and curve *d*: spin-2.

bivalued *click–click* outcomes only for the anti-parallel orientation in this limit. However, for the spin projections ($\lambda_a = s, \lambda_b = -s$) we have (see figures 2(*a*) and (*c*))

$$\mathcal{P}^s_{s-s}(1|1) = \begin{cases} 0 & \text{for all angles } \theta_{ab} \neq 0^\circ, \\ 1 & \text{for } \theta_{ab} = 0^\circ, \end{cases}$$

$$\mathcal{P}^s_{s-s}(0|1) = \begin{cases} 1 & \text{for all angles } \theta_{ab} \neq 0^\circ, \\ 0 & \text{for } \theta_{ab} = 0^\circ; \end{cases}$$

in the classical limit $s \to \infty$. In other words, when particle 2 registers a *click* in the channel $\lambda_b = -s$, the *no-click* (*click*) outcome is *always* (*never*) realized for particle 1 in the channel $\lambda_a = s$, for all the range of angles except for $\theta_{ab} = 0^\circ$. But for parallel analyser orientations $\theta_{ab} = 0^\circ$ one realizes that there is *always* a *click* outcome and *never* a *no-click* outcome for particle 1 (under the condition that particle 2 has registered a *click* result), revealing a perfect correlation between the particles in the classical limit.

In the limit $s \to \infty$ the conditional probabilities in the $\lambda_a = s, \lambda_b = 0$ channels are given by (see figure 3)

$$\mathcal{P}^s_{s0}(0|1) = 1 = \mathcal{P}^s_{s0}(0|0) \qquad \mathcal{P}^s_{s0}(1|0) = 0 = \mathcal{P}^s_{s0}(1|1)$$

i.e., irrespective of the 'yes' and 'no' conditional outcomes in the channel $\lambda_b = 0$ there is a definite 'no' answer in the $\lambda_a = s$ channel for *all* the analyser orientations θ_{ab}. In other words,

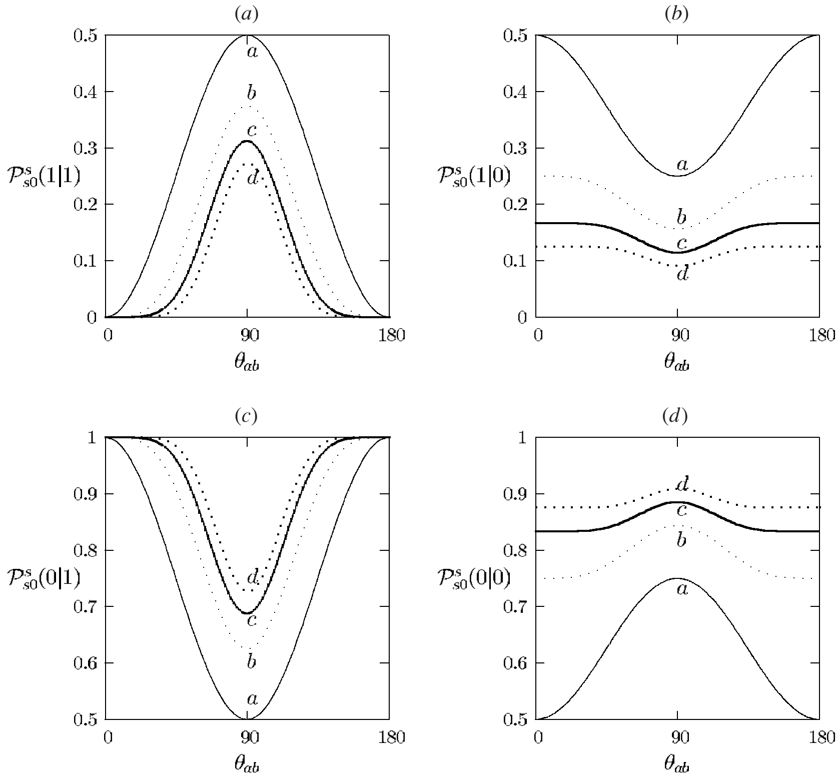

Figure 3. Conditional probabilities of the bivalued outcomes in the spin projection channels $\lambda_a = s, \lambda_b = 0$, as a function of the analyser orientation angle θ_{ab} for different spin values. Curve *a*: spin-1, curve *b*: spin-2, curve *c*: spin-3 and curve *d*: spin-4.

one can *never* find particle 1 with spin projection $\lambda_b = s$ when particle 2 is found in $\lambda_a = 0$, in the classical limit $s \to \infty$. (Note that for finite spin values, this result is not valid and the conditional probabilities in figure 3 are non-local functions of the analyser orientations θ_{ab}.)

In the limit $s \to \infty$, the above results reflect the perfect correlation between pairs of classical anti-parallel angular momentum vectors and restore the validity of local hidden variable theories for the analysis of *click–no-click* outcomes in the classical limit.

3. Information theoretic aspects

Considerable interest has been evinced recently [3, 7, 9, 14–17] in investigating quantum correlations using information theory. In this section we intend to examine how much information is contained in the bivalued *click–no-click*-measurements in various spin projection channels.

We construct Gibbs–Shannon joint information entropies (in bits) [3, 7] using the bivalued probabilities derived in section 2:

$$H\big(\hat{P}^1_{\lambda_a}(\vec{a}), \hat{P}^2_{\lambda_b}(\vec{b})\big) = -\sum_{\varepsilon_a,\varepsilon_b=0,1} \mathcal{P}^s_{\lambda_a\lambda_b}(\varepsilon_a,\varepsilon_b)\log_2 \mathcal{P}^s_{\lambda_a\lambda_b}(\varepsilon_a,\varepsilon_b). \tag{19}$$

The information contents $H\left(\hat{P}^1_{\lambda_a}(\vec{a})\right)$ and $H\big(\hat{P}^2_{\lambda_b}(\vec{b})\big)$ carried independently by the observables $\hat{P}^1_{\lambda_a}(\vec{a})$, $\hat{P}^2_{\lambda_b}(\vec{b})$ are given by

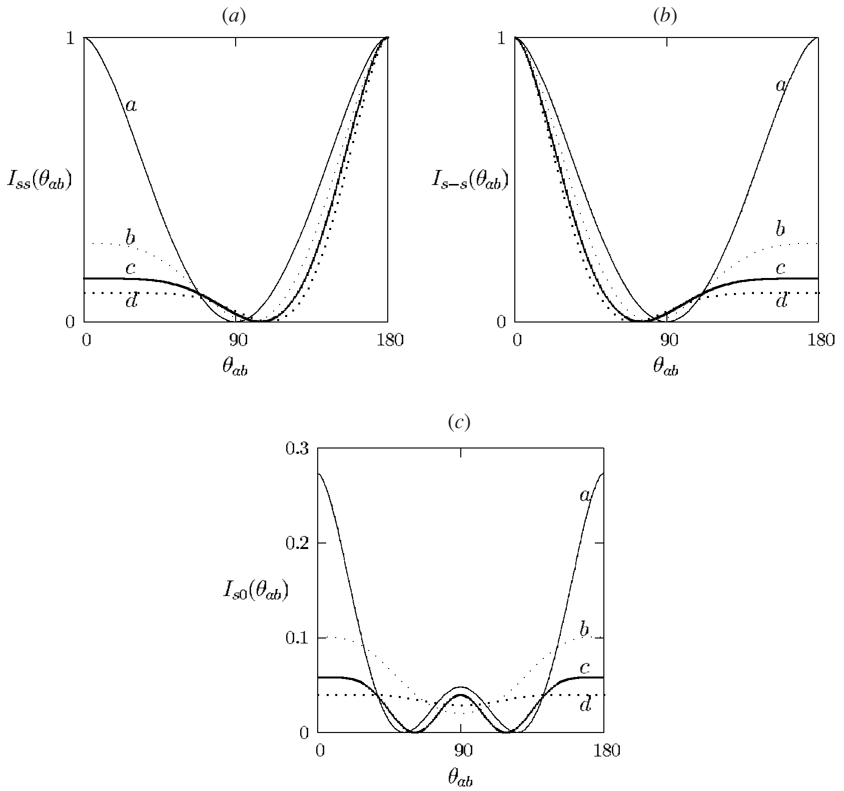

Figure 4. Information theoretic index of correlation as a function of the analyser orientation angle θ_{ab} for different values of spin. In (*a*) and (*b*), curve *a*: spin-$\frac{1}{2}$, curve *b*: spin-1, curve *c*: spin-$\frac{3}{2}$ and curve *d*: spin-2. In (*c*), curve *a*: spin-1, curve *b*: spin-2, curve *c*: spin-3 and curve *d*: spin-4. The detection channels are confined to $\lambda_a = s$, $\lambda_b = -s$ in (*b*) and $\lambda_a = s$, $\lambda_b = 0$ in (*c*).

$$
\begin{aligned}
H\left(\hat{P}^1_{\lambda_a}(\vec{a})\right) &= -\sum_{\varepsilon_a=0,1} \mathcal{P}^s_{\lambda_a}(\varepsilon_a)\log_2 \mathcal{P}^s_{\lambda_a}(\varepsilon_a) = -\frac{1}{2s+1}\log_2 \frac{(2s)^{2s}}{(2s+1)^{2s+1}}, \\
H\left(\hat{P}^2_{\lambda_b}(\vec{b})\right) &= -\sum_{\varepsilon_b=0,1} \mathcal{P}^s_{\lambda_b}(\varepsilon_b)\log_2 \mathcal{P}^s_{\lambda_b}(\varepsilon_b) = -\frac{1}{2s+1}\log_2 \frac{(2s)^{2s}}{(2s+1)^{2s+1}},
\end{aligned}
\tag{20}
$$

where we have used equation (14) for marginal probabilities $\mathcal{P}^s_{\lambda_a}(\varepsilon_a)$ and $\mathcal{P}^s_{\lambda_b}(\varepsilon_b)$. Note that these marginal information do not depend on the analyser orientations and give rise to the same information in all the spin projection channels.

The conditional information entropy $H\left(\hat{P}^1_{\lambda_a}(\vec{a})\middle|\hat{P}^2_{\lambda_b}(\vec{b})\right)$ gives the information carried by the random *click–no-click* results (i.e., measurement of the projection operator $\hat{P}^1_{\lambda_a}(\vec{a})$) under the condition that a *click* (*no-click*) outcome has occurred in the measurement of $\hat{P}^2_{\lambda_b}(\vec{b})$, and is defined through

$$
\begin{aligned}
H\left(\hat{P}^1_{\lambda_a}(\vec{a})\middle|\hat{P}^2_{\lambda_b}(\vec{b})\right) &= -\sum_{\varepsilon_a,\varepsilon_b=0,1} \mathcal{P}^s_{\lambda_a\lambda_b}(\varepsilon_a,\varepsilon_b)\log_2 \mathcal{P}^s_{\lambda_a\lambda_b}(\varepsilon_a|\varepsilon_b) \\
&= H\left(\hat{P}^1_{\lambda_a}(\vec{a}), \hat{P}^2_{\lambda_b}(\vec{b})\right) - H\left(\hat{P}^2_{\lambda_b}(\vec{b})\right).
\end{aligned}
\tag{21}
$$

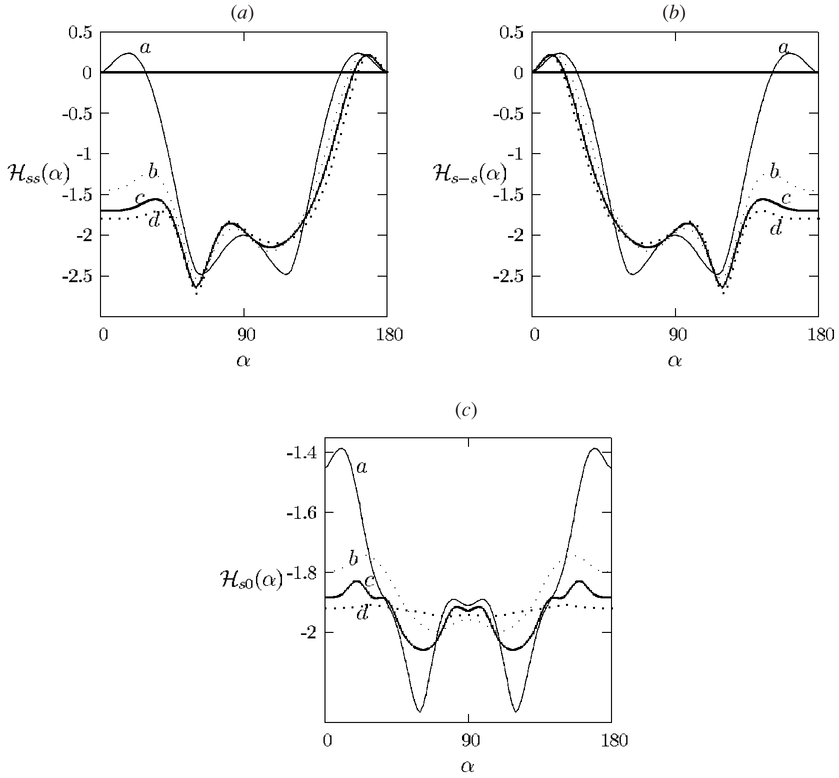

Figure 5. Information difference $\mathcal{H}(\alpha)$ for different values of spin. Curve *a*: spin-$\frac{1}{2}$, curve *b*: spin-1, curve *c*: spin-$\frac{3}{2}$ and curve *d*: spin-2 in (*a*), (*b*) while curve *a*: spin-1, curve *b*: spin-2, curve *c*: spin-3 and curve *d*: spin-4 in (*c*).

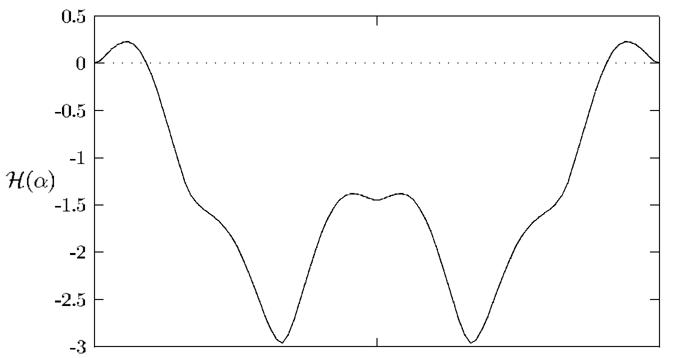

Figure 6. Information difference $\mathcal{H}(\alpha)$ for spin-1 bivalued outcomes from the detection channels $\lambda_a = 0, \lambda_b = 0$.

The mutual information entropy $I\big(\hat{P}^1_{\lambda_a}(\vec{a}), \hat{P}^2_{\lambda_b}(\vec{b})\big)$, i.e., the average information carried in common by the measurements of $\hat{P}^1_{\lambda_a}(\vec{a})$ and $\hat{P}^2_{\lambda_b}(\vec{b})$, is given by

$$I\big(\hat{P}^1_{\lambda_a}(\vec{a}), \hat{P}^2_{\lambda_b}(\vec{b})\big) = H\big(\hat{P}^1_{\lambda_a}\big) + H\big(\hat{P}^2_{\lambda_b}\big) - H\big(\hat{P}^1_{\lambda_a}(\vec{a}), \hat{P}^2_{\lambda_b}(\vec{b})\big). \quad (22)$$

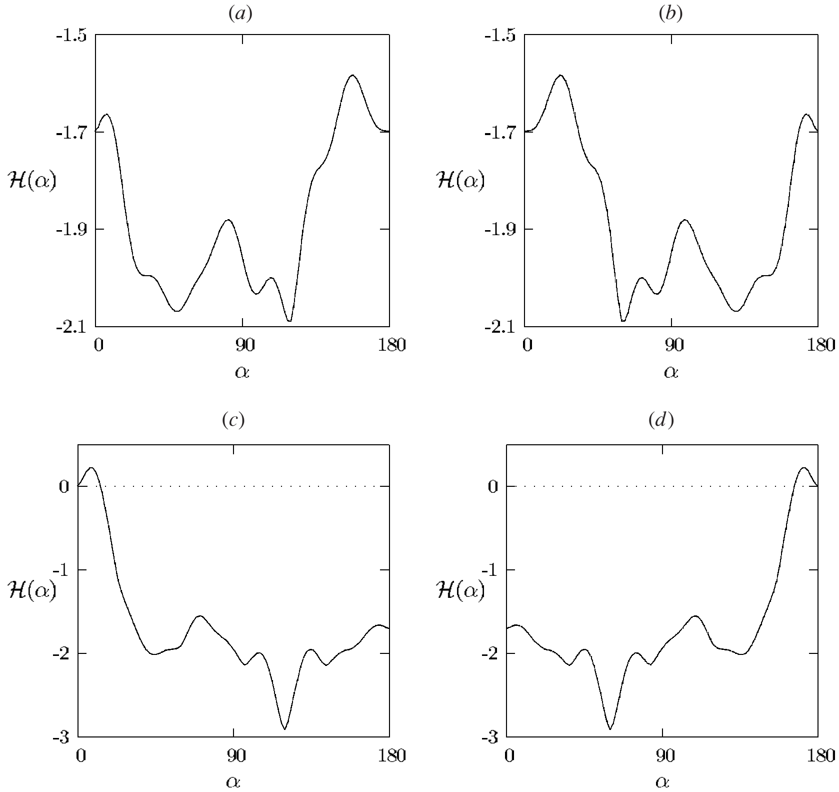

Figure 7. Information difference $\mathcal{H}(\alpha)$ for spin-$\frac{3}{2}$ bivalued outcomes resulting from different detection channels: in (*a*) $\lambda_a = \frac{3}{2}, \lambda_b = \frac{1}{2}$, in (*b*) $\lambda_a = \frac{3}{2}, \lambda_b = -\frac{1}{2}$, in (*c*) $\lambda_a = \frac{1}{2}, \lambda_b = \frac{1}{2}$ and in (*d*) $\lambda_a = \frac{1}{2}, \lambda_b = -\frac{1}{2}$.

Since the information entropies depend only on the angle θ_{ab} between the analyser orientations, we will suppress writing $\vec{a}$ and $\vec{b}$ in these quantities from now on.

It has been realized [7, 8, 15] that the mutual information entropy serves as an information theoretic index of correlation. In order to construct a normalized information theoretic index of correlation based on our bivalued outcomes, we note that the joint information entropy defined in equation (19) satisfies the *modified* [7] Araki–Lieb inequality[6],

$$H\left(\hat{P}^1_{\lambda_a}\right)\left(\text{or } H\left(\hat{P}^2_{\lambda_b}\right)\right) \leqslant H\left(\hat{P}^1_{\lambda_a}, \hat{P}^2_{\lambda_b}; \theta_{ab}\right) \leqslant H\left(\hat{P}^1_{\lambda_a}\right) + H\left(\hat{P}^2_{\lambda_b}\right), \tag{23}$$

which leads to the following bounds for the mutual information entropy:

$$0 \leqslant I\left(\hat{P}^1_{\lambda_a}, \hat{P}^2_{\lambda_b}; \theta_{ab}\right) \leqslant -\frac{1}{2s+1}\log_2\frac{(2s)^{2s}}{(2s+1)^{2s+1}}, \tag{24}$$

by making use of the explicit form for marginal information given by equation (20). Thus, an index of correlation $0 \leqslant I_{\lambda_a\lambda_b}(\theta_{ab}) \leqslant 1$ may be defined by normalizing the mutual information entropy:

$$I_{\lambda_a\lambda_b}(\theta_{ab}) = \frac{I\left(\hat{P}^1_{\lambda_a}, \hat{P}^2_{\lambda_b}; \theta_{ab}\right)}{-\frac{1}{2s+1}\log_2\frac{(2s)^{2s}}{(2s+1)^{2s+1}}} = 2 + \frac{H\left(\hat{P}^1_{\lambda_a}, \hat{P}^2_{\lambda_b}; \theta_{ab}\right)}{\frac{1}{2s+1}\log_2\frac{(2s)^{2s}}{(2s+1)^{2s+1}}}. \tag{25}$$

[6] In the case of measurements of quantum observables $\hat{A}$ and $\hat{B}$, belonging to subsystems 1 and 2 of an entangled system, the information entropy $H(\hat{A}, \hat{B})$ satisfies [7] the modified Araki–Lieb inequality: $\max(H(\hat{A}), H(\hat{B})) \leqslant H(\hat{A}, \hat{B}) \leqslant H(\hat{A}) + H(\hat{B})$. In our case, $H(\hat{A}) = H(\hat{B})$ and the inequality gets simplified further.

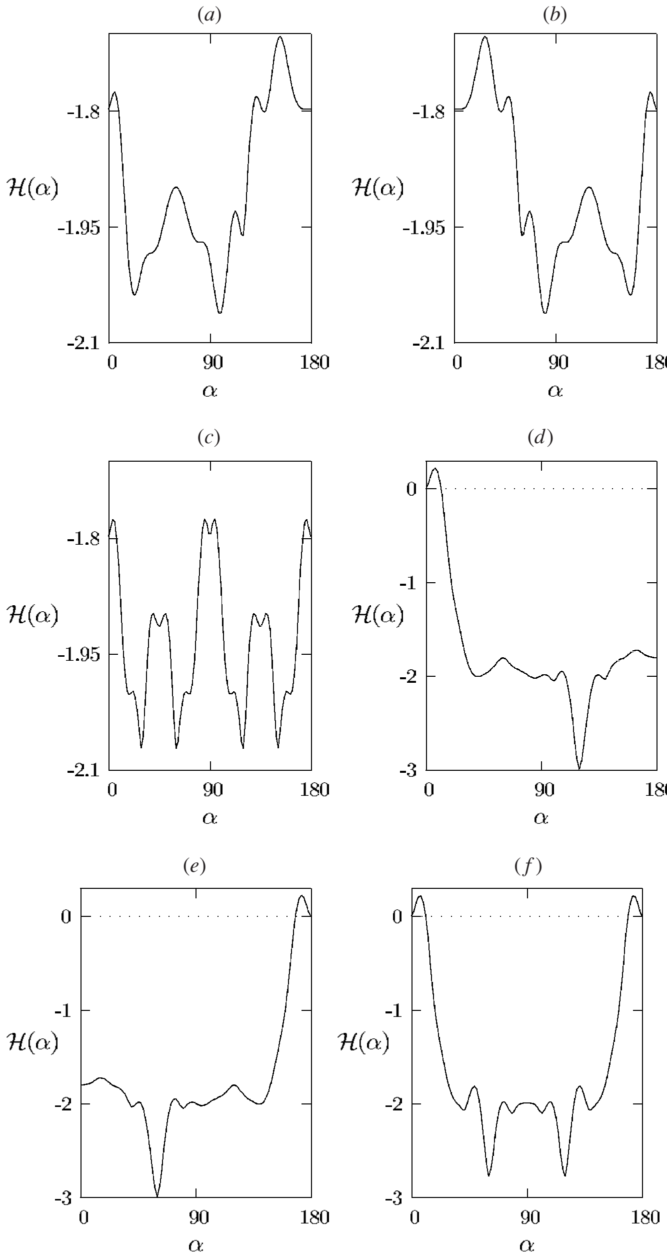

Figure 8. Information difference $\mathcal{H}(\alpha)$ for spin-2 bivalued outcomes resulting from different detection channels: in (*a*) $\lambda_a = 2, \lambda_b = 1$; in (*b*) $\lambda_a = 2, \lambda_b = -1$; in (*c*) $\lambda_a = 1, \lambda_b = 0$; in (*d*) $\lambda_a = 1, \lambda_b = 1$; in (*e*) $\lambda_a = 1, \lambda_b = -1$ and in (*f*) $\lambda_a = 0, \lambda_b = 0$.

In figure 4 we have plotted the correlation index $I_{s\pm s}(\theta_{ab})$ for the spin values $s = \frac{1}{2}, 1, \frac{3}{2}, 2$ and $I_{s0}(\theta_{ab})$ for $s = 1, 2, 3, 4$.

It could readily be observed that in the $\lambda_a = s, \lambda_b = s$ channel, the correlations between the *click–no-click* outcomes grow stronger—with the correlation index approaching the maximum value 1—as $\theta_{ab} \to 180^\circ$, i.e., when the orientation of the analysers becomes

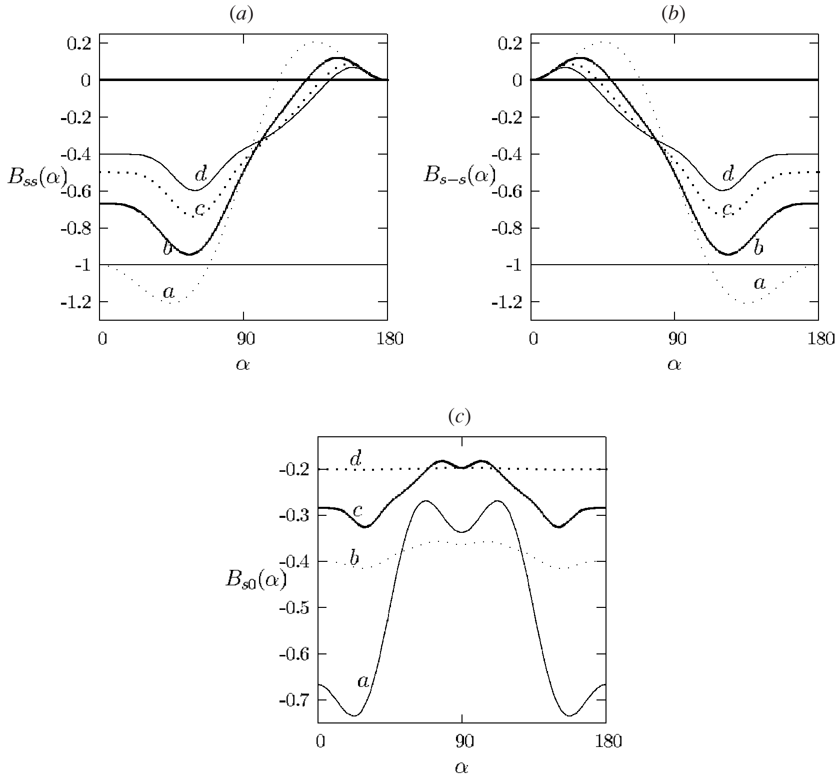

Figure 9. The Bell difference $B(\alpha)$ of spin transmissions for different values of spin. Curve *a*: spin-$\frac{1}{2}$, curve *b*: spin-1, curve *c*: spin-$\frac{3}{2}$ and curve *d*: spin-2 in (*a*), (*b*), while curve *a*: spin-1, curve *b*: spin-2, curve *c*: spin-3 and curve *d*: spin-4 in (*c*) and (*d*).

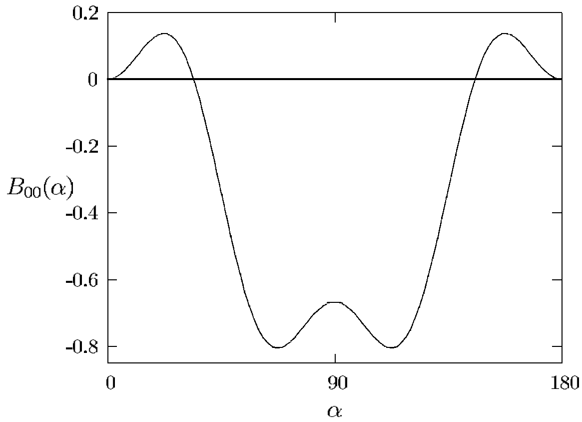

Figure 10. Bell difference $B(\alpha)$ for spin-1 bivalued outcomes from the detection channels $\lambda_a = 0, \lambda_b = 0$.

nearly anti-parallel, whereas in the $\lambda_a = s$, $\lambda_b = -s$ channel, the correlation index attains its maximum value 1 for $\theta_{ab} = 0°$, i.e, when the analysers are parallel. Note that the index of correlation is less than 0.3 and indicates that the correlation between the bivalued outcomes in the $\lambda_a = s$, $\lambda_b = 0$ channel is quite poor.

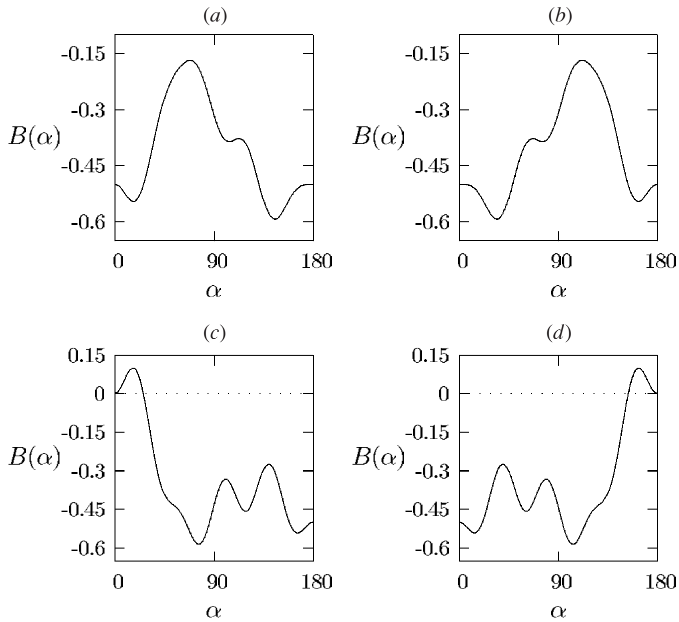

Figure 11. Spin transmission difference $B(\alpha)$ for spin-$\frac{3}{2}$ bivalued outcomes resulting from different detection channels: In (*a*) $\lambda_a = \frac{3}{2}, \lambda_b = \frac{1}{2}$, in (*b*) $\lambda_a = \frac{3}{2}, \lambda_b = -\frac{1}{2}$, in (*c*) $\lambda_a = \frac{1}{2}, \lambda_b = \frac{1}{2}$ and in (*d*) $\lambda_a = \frac{1}{2}, \lambda_b = -\frac{1}{2}$.

Information theoretic Braunstein–Caves (BC) inequalities [9] involve conditional information entropies $H(\hat{A}|\hat{B})$ and dictate that the subsystem observables (denoted by $\hat{A}$ and $\hat{B}$) of any entangled system can carry information in accordance with

$$H(\hat{A}|\hat{B}) \leqslant H(\hat{A}|\hat{B}') + H(\hat{A}'|\hat{B}') + H(\hat{A}'|\hat{B}), \tag{26}$$

so as to be consistent with local realistic theories. It has been observed [9] that correlations between the spin components $\hat{\vec{S}}_1 \cdot \vec{a}, \hat{\vec{S}}_2 \cdot \vec{b}$ of subsystems 1 and 2 of a EPRB spin singlet state violate BC inequalities. We would like to investigate the *click–no-click* outcomes in various possible spin projection channels with the intention of isolating the correlations that lead to violation of the BC inequality.

For the bivalued outcomes resulting from the spin projection channels λ_a and λ_b the BC inequality has the form

$$H_{\lambda_a\lambda_b}(\theta_{ab}) \leqslant H_{\lambda_a\lambda_b}(\theta_{ab'}) + H_{\lambda_a\lambda_b}(\theta_{a'b'}) + H_{\lambda_a\lambda_b}(\theta_{a'b}), \tag{27}$$

where we have expressed the conditional information entropies through $H\big(\hat{P}^1_{\lambda_a}\big|\ \hat{P}^2_{\lambda_b}; \theta_{ab}\big) \equiv H_{\lambda_a\lambda_b}(\theta_{ab})$ for simplicity. Here, the angles $\theta_{ab} = \cos^{-1}(\vec{a}\cdot\vec{b}), \theta_{ab'} = \cos^{-1}(\vec{a}\cdot\vec{b}'), \theta_{a'b'} = \cos^{-1}(\vec{a}'\cdot\vec{b}')$ and $\theta_{a'b} = \cos^{-1}(\vec{a}'\cdot\vec{b})$ correspond to four different orientations of the analysers. For the special case when the successive vectors $\vec{a}, \vec{b}', \vec{a}', \vec{b}$ are coplanar and are separated by an angle α (i.e., $\vec{a}\cdot\vec{b}' = \vec{a}'\cdot\vec{b}' = \vec{a}'\cdot\vec{b} = \cos\alpha$ and $\vec{a}\cdot\vec{b} = \cos 3\alpha$), the BC inequality is violated if

$$\mathcal{H}_{\lambda_a\lambda_b}(\alpha) \equiv H_{\lambda_a\lambda_b}(3\alpha) - 3H_{\lambda_a\lambda_b}(\alpha) \tag{28}$$

is positive. We have plotted $\mathcal{H}_{s\pm s}(\alpha)$ for spin values $s = \frac{1}{2}, 1, \frac{3}{2}, 2$ and $\mathcal{H}_{s0}(\alpha)$ for $s = 1, 2, 3, 4$ in figure 5. We note that the bivalued correlations in the $\lambda_a = s, \lambda_b = 0$ channels do not violate the BC inequality. To get further insight into this observation we have plotted $\mathcal{H}_{\lambda_a\lambda_b}(\alpha)$ for all the channels—other than the ones already considered in figures 5—when $s = 1, \frac{3}{2}, 2,$

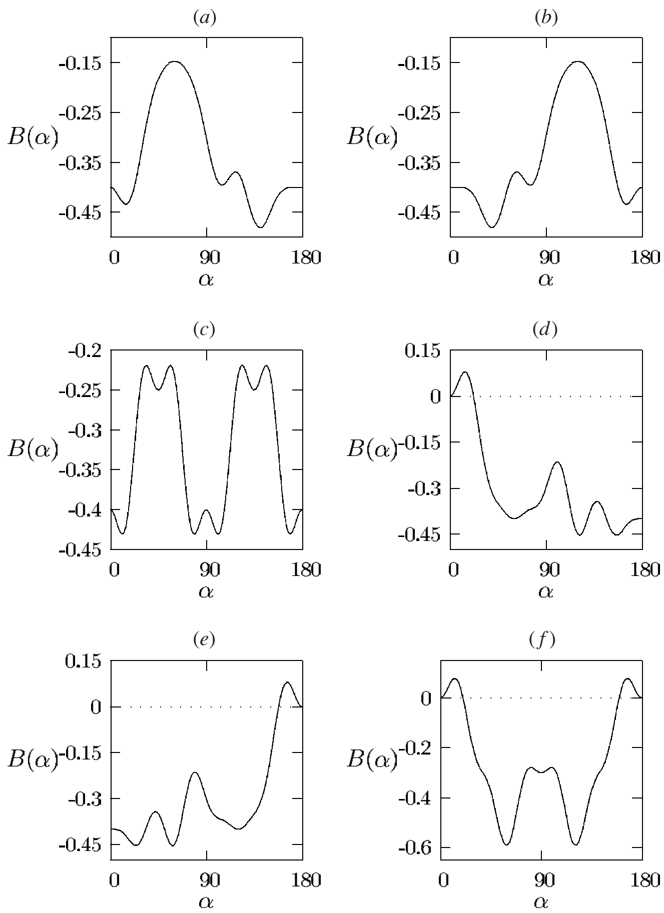

Figure 12. The Bell difference $B(\alpha)$ for spin-2 bivalued outcomes resulting from different detection channels: in (*a*) $\lambda_a = 2, \lambda_b = 1$; in (*b*) $\lambda_a = 2, \lambda_b = -1$; in (*c*) $\lambda_a = 1, \lambda_b = 0$; in (*d*) $\lambda_a = 1, \lambda_b = 1$; in (*e*) $\lambda_a = 1, \lambda_b = -1$ and in (*f*) $\lambda_a = 0, \lambda_b = 0$.

in figures 6–8. We note that the BC inequalities are violated only when $|\lambda_a| = |\lambda_b|$. In other words, the bivalued correlations in the $|\lambda_a| \neq |\lambda_b|$ channels appear to be classical in nature.

Local realistic theories predict that the spin transmissions satisfy the Bell type inequality given by equation (2) and it has been shown [5] that this inequality is violated for all values of s, even though the strength of violation reduces with increasing spin value s. We identify that the violation of inequality (2) is verified only for spin transmissions in the *maximum spin down* channels. Noting that the quantum mechanical spin transmissions of equation (1) are given by

$$\begin{aligned} p(\vec{a}, \vec{b}) &= \sum_{\varepsilon_a,\varepsilon_b=0,1} \mathcal{P}^s_{\lambda_a\lambda_b}(\varepsilon_a, \varepsilon_b)\varepsilon_a\varepsilon_b = \mathcal{P}^s_{\lambda_a\lambda_b}(1, 1), \\ p(\vec{a}) &= \sum_{\varepsilon_a,\varepsilon_b=0,1} \mathcal{P}^s_{\lambda_a\lambda_b}(\varepsilon_a, \varepsilon_b)\varepsilon_a = \mathcal{P}^s_{\lambda_a}(1) = \frac{1}{2s+1} \\ p(\vec{b}) &= \sum_{\varepsilon_a,\varepsilon_b=0,1} \mathcal{P}^s_{\lambda_a\lambda_b}(\varepsilon_a, \varepsilon_b)\varepsilon_b = \mathcal{P}^s_{\lambda_b}(1) = \frac{1}{2s+1}, \end{aligned} \qquad (29)$$

we are in a position to explore the validity of the Bell inequality (2) for quantum mechanical spin transmissions in various possible channels λ_a, λ_b, $(-s \leqslant \lambda_a, \lambda_b \leqslant s)$. For the co-planar geometry ($\vec{a} \cdot \vec{b}' = \vec{a}' \cdot \vec{b}' = \vec{a}' \cdot \vec{b} = \cos\alpha$ and $\vec{a} \cdot \vec{b} = \cos 3\alpha$) we express the Bell difference of spin transmissions as

$$B^{s}_{\lambda_a\lambda_b}(\alpha) = 3p(\alpha) - p(3\alpha) - \frac{2}{2s+1}, \tag{30}$$

so that inequality (2) assumes the form

$$-1 \leqslant B^{s}_{\lambda_a\lambda_b}(\alpha) \leqslant 0. \tag{31}$$

In figures 9–12 we have plotted the Bell difference $B^{s}_{\lambda_a\lambda_b}(\alpha)$ of spin transmissions in various channels we had analysed in connection with the information theoretic co-planar BC inequalities. Interestingly, the violation of spin transmission Bell inequalities indeed support the result realized through the information theoretic BC inequalities, namely, *only the spin channels* $|\lambda_a| = |\lambda_b|$ *record non-classical correlations*. In this regard, therefore, the information theoretic BC inequality (27) and the Bell inequality (2) are equivalent[7].

4. Conclusion

We have generalized the work of Wódkiewicz [3] and studied the bivalued *click* or *no-click* outcomes—resulting from a EPRB spin-s singlet state—in different spin projection channels λ_a, λ_b. This study provides a framework for the analysis of non-locality underlying EPRB correlations. In the case of observations in maximum *spin down* channels, we have shown that our results agree with those of Wódkiewicz [3]. We have analysed the information theoretic aspects of *click–no-click* results using Gibbs–Shannon entropies. This leads us to the observation that the coplanar BC inequalities are violated for the bivalued correlations only in the $|\lambda_a| = |\lambda_b|$ channels. We have also verified that the violation of the coplanar spin transmission Bell inequality indeed reflects the non-classical nature of the bivalued outcomes in the spin channels $|\lambda_a| = |\lambda_b|$.

Acknowledgments

We thank the referees for their useful suggestions in the light of which the paper is revised.

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[7] Both these inequalities involve statistical averages of spin irreducible tensors of higher ranks (i.e., higher order spin alignments) [5] not only averages of vector operators that are involved in the standard spin correlation Bell inequalities, valid for spin $\frac{1}{2}$ particles.

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